Common Core Standard:
7.G.2

Construct Geometric Shapes

Getting the Idea

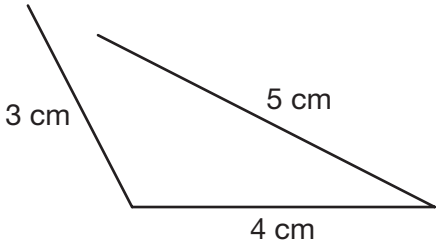
Triangles can be constructed using simple tools such as a ruler and protractor, or using more complex tools such as computer drawing technology. When drawing triangles with a ruler and/or a protractor by hand, you may sometimes need to erase your work and start over again. It can involve some trial and error.

Example 1

Using a ruler, construct a triangle with side lengths of 3 centimeters, 4 centimeters, and 5 centimeters. What kind of triangle is it? Is it possible to draw another kind of triangle?

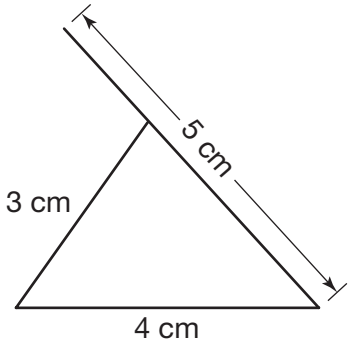
Strategy **Use a ruler.**

Step 1 Try drawing a triangle with one obtuse angle—an obtuse triangle.



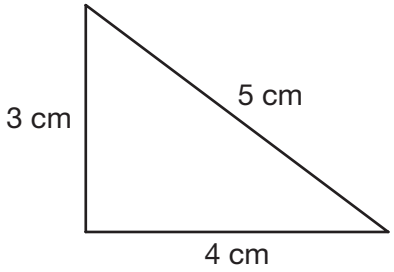
The figure is not closed. An obtuse triangle is not possible.

Step 2 Try drawing a triangle with only acute angles—an acute triangle.



The endpoints do not meet. An acute triangle is not possible.

Step 3 Try drawing a triangle with one right angle—a right triangle.



You can draw one unique triangle with those side lengths, and it is a right triangle.

Solution **It is only possible to draw a right triangle with side lengths of 3 centimeters, 4 centimeters, and 5 centimeters.**

When trying to construct a triangle with given side lengths or angle measures, there are several possibilities:

- The triangle may be uniquely defined. In other words, you may only be able to draw one triangle.
- The triangle may be ambiguously defined. That just means you may be able to draw more than one triangle.
- The triangle may be nonexistent. It may not be possible to draw a triangle with those measures.

The triangle in Example 1 is uniquely defined.

Below is a theorem that can help you construct triangles with given angle measures.

Triangle Angle Sum Theorem

This theorem states that the sum of the angles in any triangle is 180°.

Example 2

Is it possible to construct a triangle with angles measuring 61°, 33°, and 86°? If so, can you draw only a unique triangle or can you draw many different triangles?

Strategy **Use the triangle angle sum theorem to test the angle measures. Then decide if it is possible to draw only a unique triangle or many different triangles.**

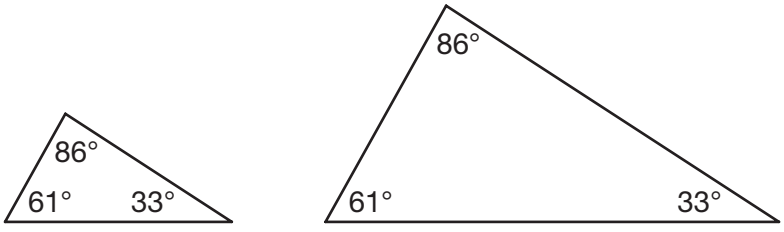
Step 1 Determine if the angle measures add to 180°.

$61° + 33° + 86° \stackrel{?}{=} 180°$

$180° = 180°$ ✓

So, a triangle with these angle measures is possible.

Step 2 Use a protractor to draw one or more triangles with those angle measures.



The two triangles above have the correct angle measures, but they have different lengths.

That is because triangles with the same angle measures are similar to one another.

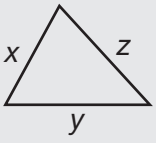
So, it is possible to draw many different triangles with those angle measures.

Solution **A triangle with angles measuring 61°, 33°, and 86° is ambiguously defined because no side lengths are mentioned. It is possible to draw many different similar triangles with those angle measures.**

Another theorem that may help you when constructing triangles is given below.

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$x + y > z$ $x + z > y$ $y + z > x$

Example 3

Can you construct a triangle with sides measuring 5 inches, 8 inches, and 15 inches?

Strategy **Use the triangle inequality theorem.**

Step 1 Use the triangle inequality theorem.

If this triangle is possible, the sum of any two side lengths should not be greater than the third side length.

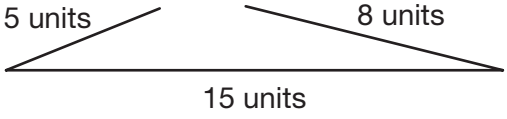
$5 + 8 \overset{?}{>} 15$?

$13 < 15$, so the inequality $5 + 8 > 15$ is not true.

It is impossible to draw a triangle with those side lengths.

Step 2 Try to sketch a triangle with those dimensions so you can see why it is not possible.

Use units other than inches to keep your drawing a manageable size.



There is no way to connect all three sides. It is impossible to draw a triangle with those side lengths.

Solution **A triangle with sides measuring 5 inches, 8 inches, and 15 inches does not exist.**

Coached Example

Is it possible to construct a triangle with sides measuring 8 feet, 9 feet, and 12 feet?

The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle must be _____ than the length of the third side.

Use the theorem to determine if this triangle is possible or not.

Is $8 + 9 > 12$? _____

Is $8 + 12 > 9$? _____

Is $9 + 12 > 8$? _____

The inequalities above are all true, so it _____ possible to draw a triangle with side lengths of 8 feet, 9 feet, and 12 feet.

Lesson Practice

Choose the correct answer.

1. Which best describes a triangle with side lengths of 6 inches, 8 inches, and 9 inches?

A. ambiguously defined
B. nonexistent
C. a unique, acute triangle
D. a unique, right triangle

2. Which best describes a triangle with angles measuring 60°, 40°, and 100°?

A. ambiguously defined
B. nonexistent
C. a unique, acute triangle
D. a unique, right triangle

3. Which best describes a triangle with side lengths of 3 inches, 4 inches, and 8 inches?

A. ambiguously defined
B. nonexistent
C. a unique, obtuse triangle
D. a unique, right triangle

4. Which best describes a triangle with side lengths of 5 centimeters, 12 centimeters, and 13 centimeters?

A. ambiguously defined
B. nonexistent
C. a unique, acute triangle
D. a unique, right triangle

5. Lincoln constructs a triangle with one side 5 inches long and another side 7 inches long. Which is **not** a possible length for the third side?

A. 3 inches
B. 6 inches
C. 11 inches
D. 12 inches

6. Maggie constructs a triangle with one side 7 centimeters long and another side 10 centimeters long. Which is **not** a possible length for the third side?

A. 2 centimeters
B. 4 centimeters
C. 7 centimeters
D. 16 centimeters

7. Chantelle constructs a triangle with angles measuring 65° and 38°. What must be true of the measure of the third angle in her construction?

A. It must measure exactly 77°.

B. It must measure exactly 87°.

C. It can have any measure less than 103°.

D. It can have any measure greater than 27°.

8. Josh constructs a triangle with angles measuring 54°, 23°, and 103°. He wants to construct a different triangle with those angle measures. What will he find if he does?

A. It is not possible to construct a different triangle with those angle measures.

B. Any other triangle he constructs with those angle measures will be congruent to his original triangle.

C. Any other triangle he constructs with those angle measures will be similar to his original triangle.

D. He can construct many other triangles with those angle measures, and none of them will be similar to the first triangle he constructed.

9. Use a ruler for this problem.

A. In the space below, construct a triangle with side lengths of 1 centimeter, 2 centimeters, and 2 centimeters.

B. Is the triangle you constructed above unique or ambiguously defined? Use words and a sketch to support your answer.

10. Select True or False for each statement.

A. An ambiguously defined triangle can have angle measures of 43°, 79°, and 58°. ○ True ○ False

B. A triangle **cannot** have angle measures of 32°, 78°, and 80°. ○ True ○ False

C. A unique, acute triangle can have angle measures of 60°, 70°, and 90°. ○ True ○ False

D. A unique, right triangle can have angle measures of 30°, 60°, and 90°. ○ True ○ False

11. Write each set of triangle side lengths in the correct box.

2 cm, 3 cm, 6 cm	4 mi, 9 mi, 13 mi
5 ft, 12 ft, 13 ft	6 in., 8 in., 10 in.
15 mm, 36 mm, 39 mm	14 yd, 16 yd, 30 yd

Unique Triangle	Nonexistent Triangle

12. Brandon drew a triangle with one side 6 centimeters long and another side 10 centimeters long. Look at each length below. Is the length a possible side length of Brandon's triangle? Select Yes or No.

A.	2 centimeters	○ Yes	○ No
B.	4 centimeters	○ Yes	○ No
C.	6 centimeters	○ Yes	○ No
D.	10 centimeters	○ Yes	○ No
E.	14 centimeters	○ Yes	○ No
F.	18 centimeters	○ Yes	○ No

13. Draw a line from each triangle description to a possible set of side lengths.

A.	nonexistent triangle	• 5 m, 18 m, 27 m
B.	unique, right triangle	• 10 ft, 10 ft, 10, ft
C.	unique, acute triangle	• 12 in., 16 in., 20 in.

14. Use angle measures from the box to complete each set of three angle measures of a triangle.

59°, 63°, _____

64°, 48°, _____

72°, 56°, _____

60°, 58°, _____

52°
58°
62°
68°